



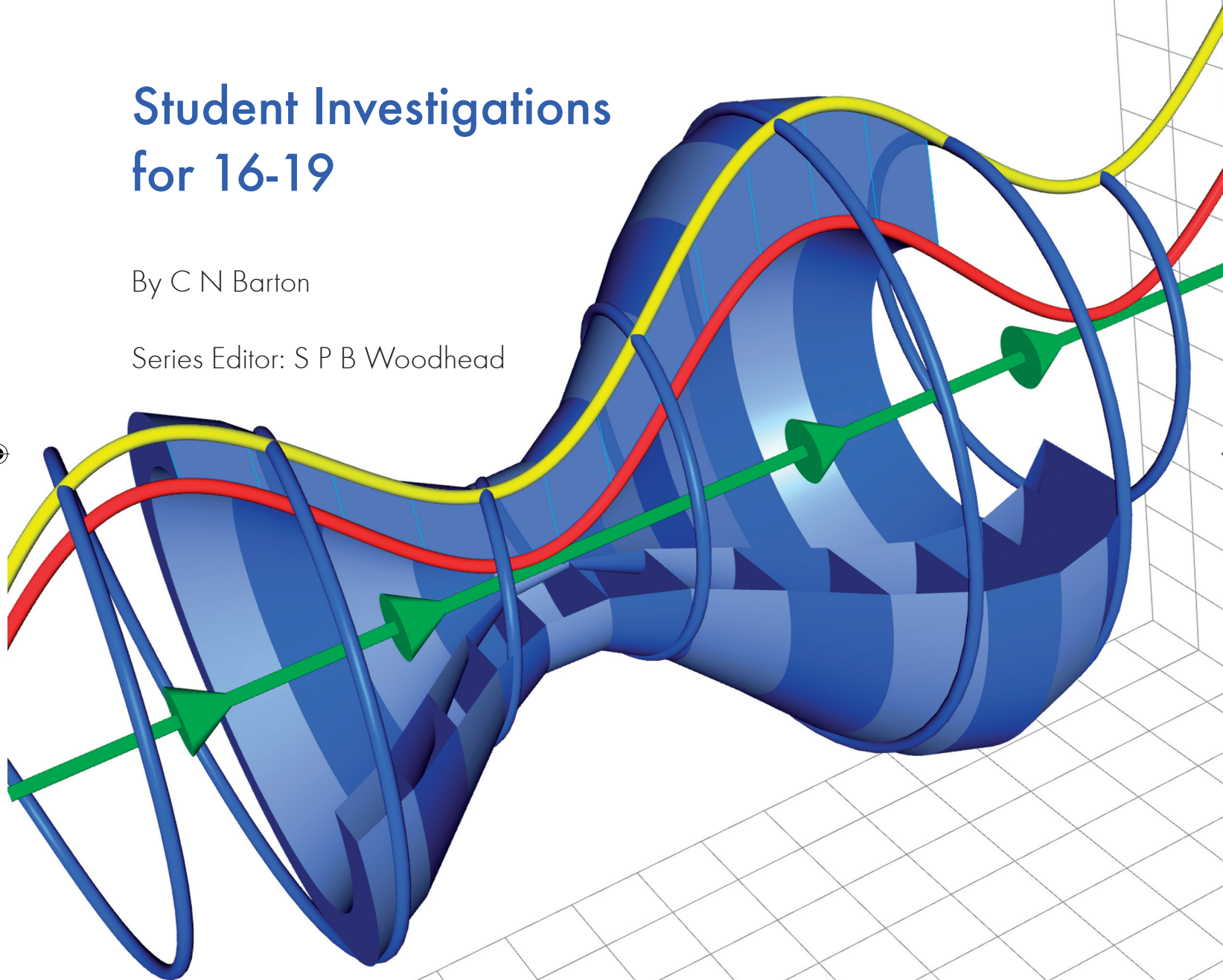
# Autograph

## *Activities*

Student Investigations  
for 16-19

By C N Barton

Series Editor: S P B Woodhead



EASTMOND  
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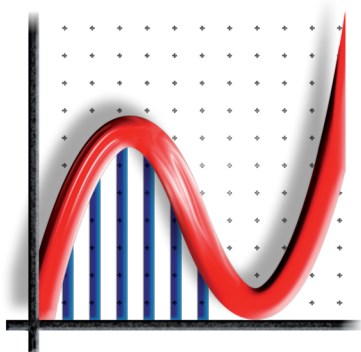
Key

**Autograph Function Name**  
**Keyboard Shortcut**



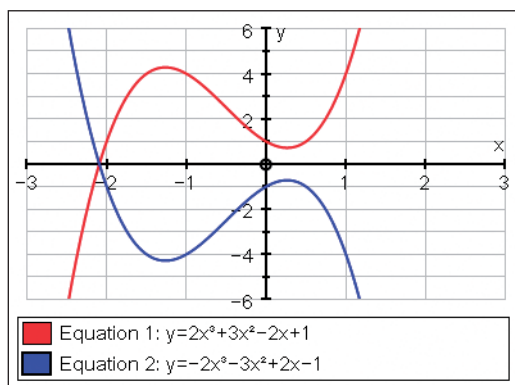
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## GETTING GOING



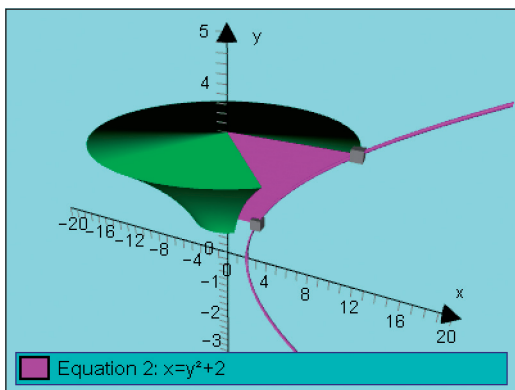
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## STUDENT INVESTIGATIONS



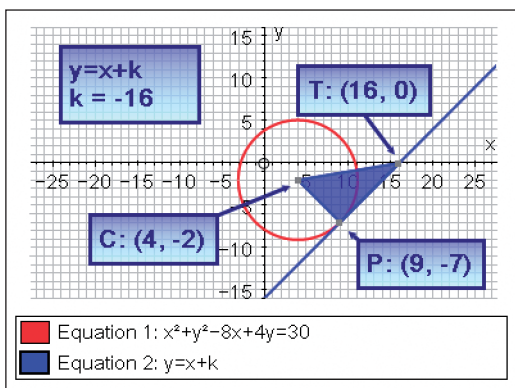
S1 Transforming the Graphs of Functions.....	41
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*In this investigation students are introduced to each of the 6 types of transformations that can be applied to a function. The use of Autograph's constant controller allows students to test their predictions and deepen their level of understanding.*



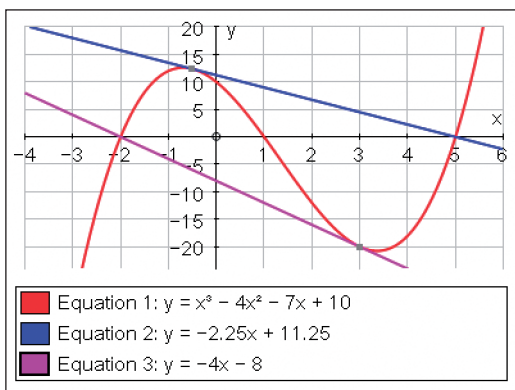
### S2 Volume of Revolution Investigation.....67

*During this dynamic and interactive investigation, students are able to use Autograph's unique 3D interface to visualise the solids formed by rotating areas around axes and lines. This should strengthen the link between the algebra and the geometry of what can otherwise be a conceptually difficult topic.*



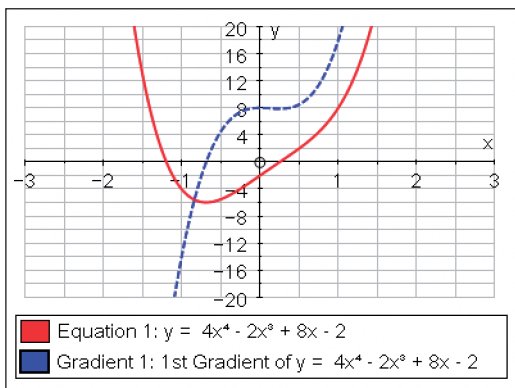
### S3 Co-ordinate Geometry Practice .....95

*This interactive investigation tests students' knowledge and understanding of all the key skills involved in co-ordinate geometry. There is plenty of scope for prediction and investigation to help consolidate knowledge and deepen understanding.*



### S4 A Cubic Investigation..... 119

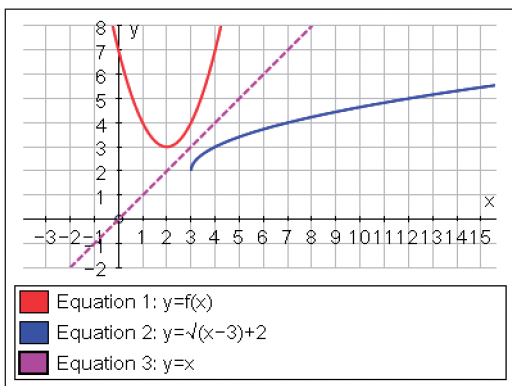
*An investigation into the unusual and interesting relationships between the roots of cubic equations, which tests students' knowledge of many important areas of co-ordinate geometry and mathematics in general.*



### S5 Discovering the Rule for the Gradient Function..... 137

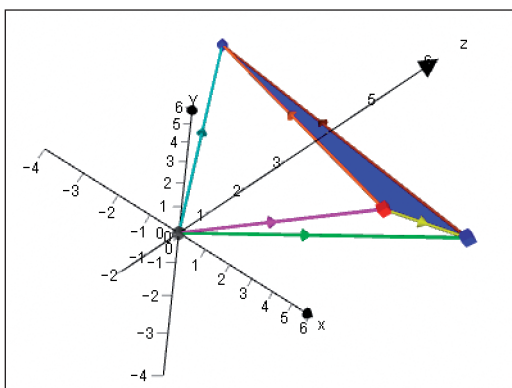
*During this investigation, students discover the rule for finding the gradient function of quadratics, cubics, and functions involving higher powers of x. They are encouraged to predict and then use Autograph to test out their predictions in a highly visual and dynamic manner, thus deepening their levels of understanding.*





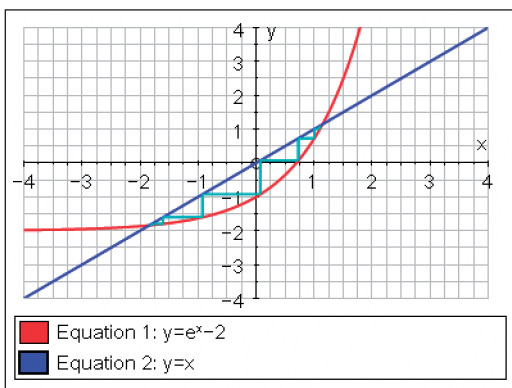
### S6 Investigating Functions..... 161

*This interactive investigation is designed to introduce students to the main concepts involved in the study of functions, including different types of functions, domain and range, and inverse functions.*



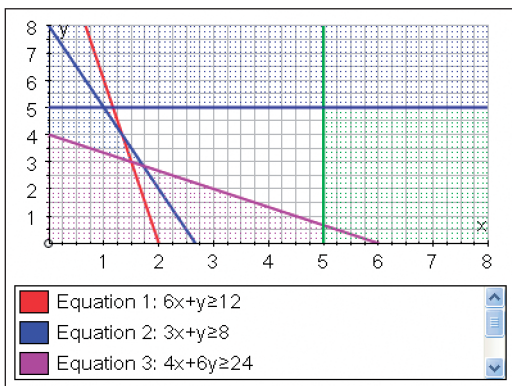
### S7 Investigating Vectors in 2D and 3D..... 187

*In this interactive investigation, students are able to consolidate, deepen and enrich their knowledge and understanding of vectors, both in 2D and 3D, using Autograph's dynamic tools and unique 3D interface.*



### S8 Investigating Iteration..... 211

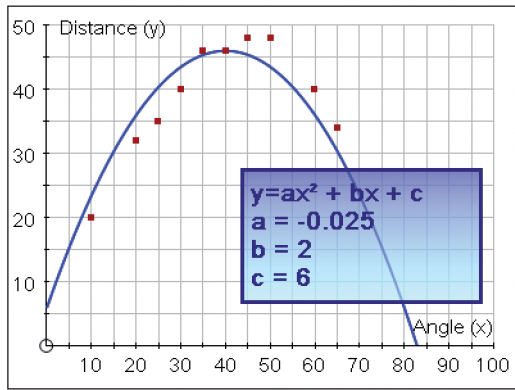
*Students can make full use of one of Autograph's iteration functions in this dynamic and interactive investigation which conveys the link between the algebraic and graphical solution to iterative equations.*



### S9 Linear Programming Investigations..... 241

*Three investigations into Linear Programming which make use of Autograph's dynamic and interactive controls to enable students to visualise the graphical solutions to such problems, and hence deepen understanding.*





S10 Flying Gonzo ..... 267

*A fun investigation into a real life application of mathematics. Students must devise a model which enables Gonzo to fly successfully from a cannon into a bucket of water. Many important mathematical concepts are incorporated along the way!*



S2

# VOLUME OF REVOLUTION INVESTIGATION

STUDENT WORKSHEET NAME:.....

## Setting up your Page

In order to take full advantage of Autograph's unique 3D world, we first need to set up our page correctly. We will do this at the start, and then use the same page for the whole of the investigation.



Open up Autograph in **Advanced Mode**.



Open up a **New 3D Graph Page**.



Make sure you are not in **Whiteboard Mode** (there should not be a blue square around the button).



Change the colour of the background to **Light**.



Change the position of the key to the right-hand side of the screen.



Click on the little arrow next to **x-y-z Orientation**.



Select **x-y Orientation**.



Edit the axes as follows:

**x:** Minimum: 0    Maximum: 4    Numbers: 1    Pips: 0.2

**y:** Minimum: -12    Maximum: 12    Numbers: 2    Pips: 1

**z:** Minimum: -12    Maximum: 12    Numbers: 2    Pips: 1

Remove all of the green ticks underneath **Auto**.

**Note:** You must ensure all the ticks under **Auto** are removed or Autograph will attempt to re-scale your axes for you.

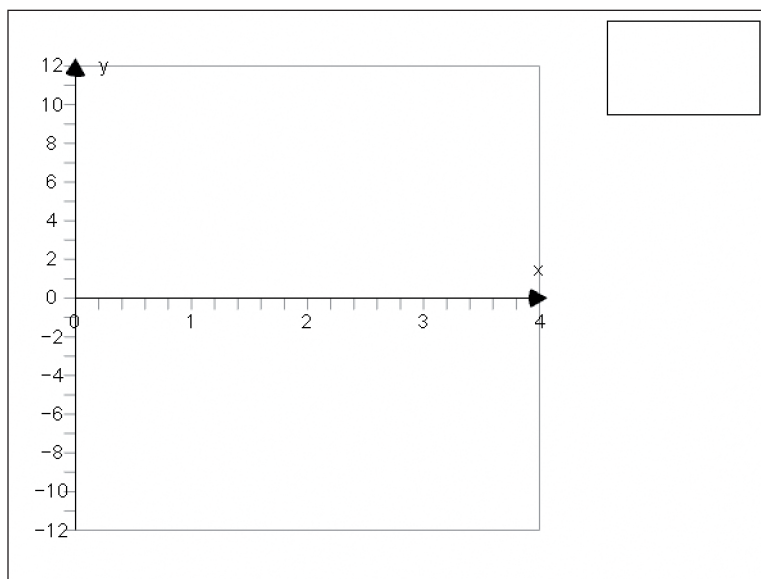


Still on the **Edit Axes** screen, click on the **Options** tab.

Under **Axes**, remove the tick next to **Always Outside**.

Click **OK**.

Your screen should hopefully look something like this:



### ACTIVITY 1



Enter the equation:  $y = ax + b$

Still on the **Add Equation** screen, place a tick in the space next to **Plot as 2D equation**.

Click on **Edit Constants** and change the value of  $a$  to 3 and the value of  $b$  to 0.

**Predict**

*What will this line look like? Where will it cross the axes?*

Click **OK** twice.



**TASK 1:** Work out the area of the triangle which would be formed by the  $x$ -axis, the line  $y = ax + b$ , and the line  $x = 3$ .

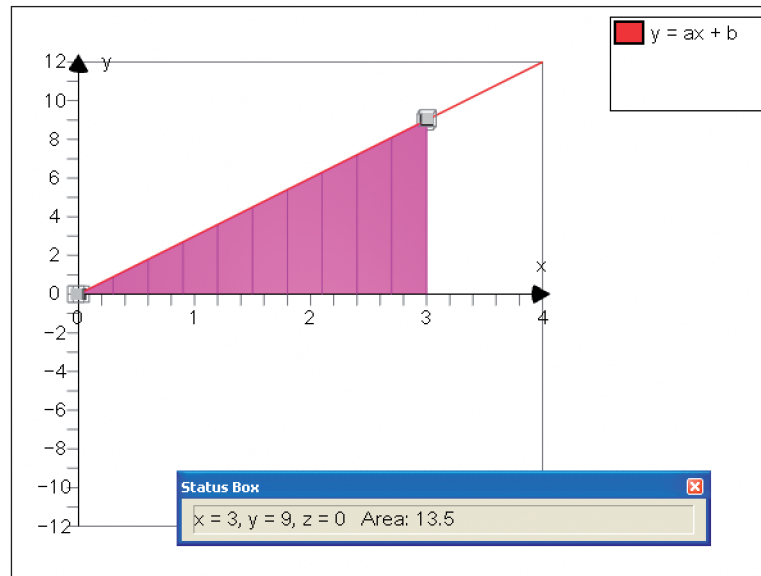
**Hint:**  $A = \frac{bh}{2}$

S2

# VOLUME OF REVOLUTION INVESTIGATION

## ANSWERS

### ACTIVITY 1



**TASK 1:** Area of triangle:

$$A = \frac{bh}{2} = \frac{3 \times 9}{2} = 13.5$$

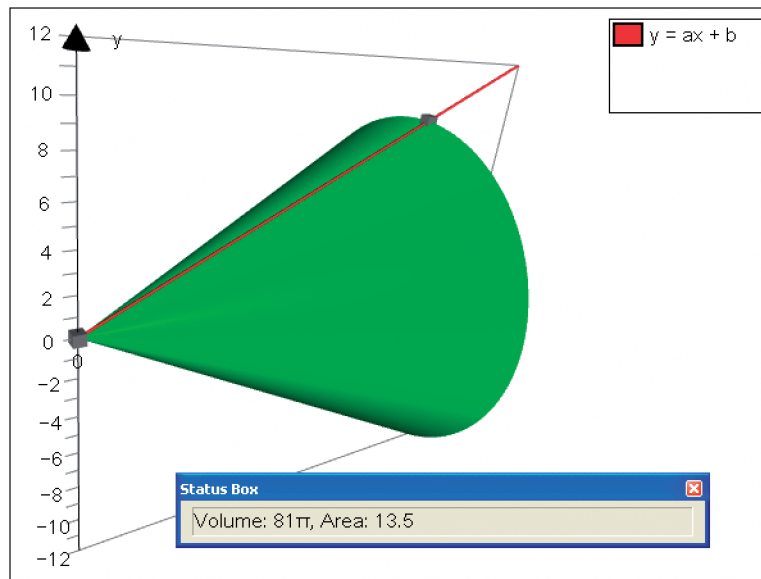
**TASK 2:** Using integration:

$$A = \int_0^3 3x \, dx = \left[ \frac{3x^2}{2} \right]_0^3 = \frac{27}{2} - 0 = 13.5$$

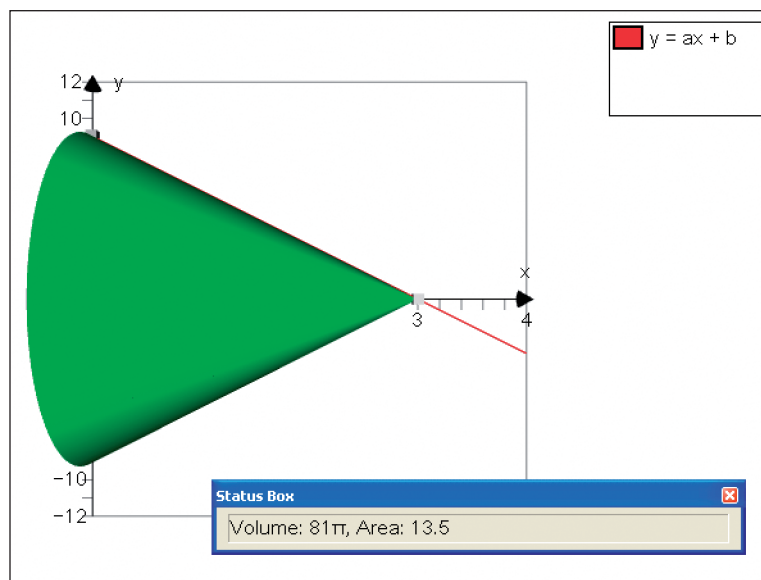
**TASK 3:** The solid formed will be a cone. The radius of the base is 9, and the height is 3.

**TASK 4:** Volume of the cone:

$$V = \pi \int_0^3 y^2 dx = \pi \int_0^3 (3x)^2 dx = \pi \int_0^3 9x^2 dx = \pi [3x^3]_0^3 = 81\pi$$



**TASK 5:** The ends of the cone would need to be switched around, so the pointed end is at (3, 0), and the flat circular face is along the y-axis. The line  $y = ax + b$  would need to have negative gradient, and cross the y-axis at (0, 9). The values of  $a$  and  $b$  which give this are  $a = -3$ ,  $b = 9$ , giving  $y = -3x + 9$ .



**TASK 6:** The solid formed will be a cylinder, with radius 4 and height/length 3.